Celerity group and wave shoaling in Boussinesq-type models

Gonzalo Simarro

(ICM–CSIC, Barcelona, Spain)

In collaboration with A. Orfila and A. Galan

September 30, 2014

Wave propagation models

ELSEVIER

Wave Motion 33 (2001) 225–243

WAVE MOTION

www.elsevier.com/locate/wavemoti

Boussinesq-type equations with improved nonlinear performance

Andrew B. Kennedy*, James T. Kirby, Qin Chen, Robert A. Dalrymple

Center for Applied Coastal Research, University of Delaware, Newark, DE 19716, USA.

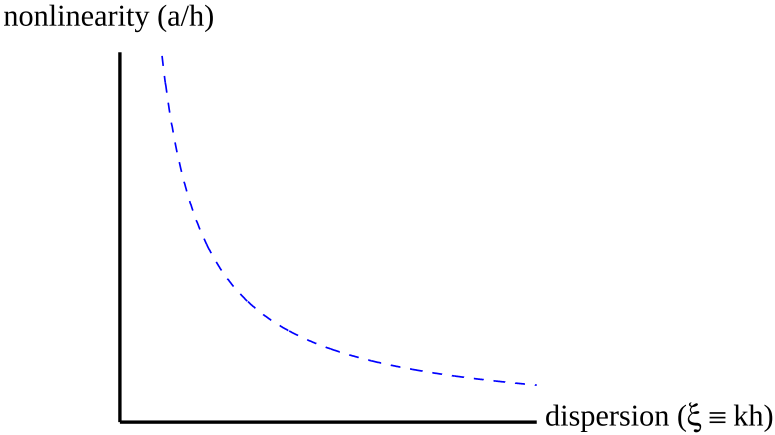
Received 19 November 1999; accepted 27 June 2000

Abstract

In this paper, we derive and test a set of extended Boussinesq equations with improved nonlinear performance. To do this, the concept of a reference elevation is further generalised to include a time-varying component that moves with the instantaneous free surface. It is found that, when compared to Stokes-type expansions of the second harmonic and fully nonlinear potential flow computations, both theoretical and practical nonlinear performance can be considerably improved. Finally, a special case of the extended equations is found to have properties which are invariant with respect to the still water datum. © 2001 Elsevier

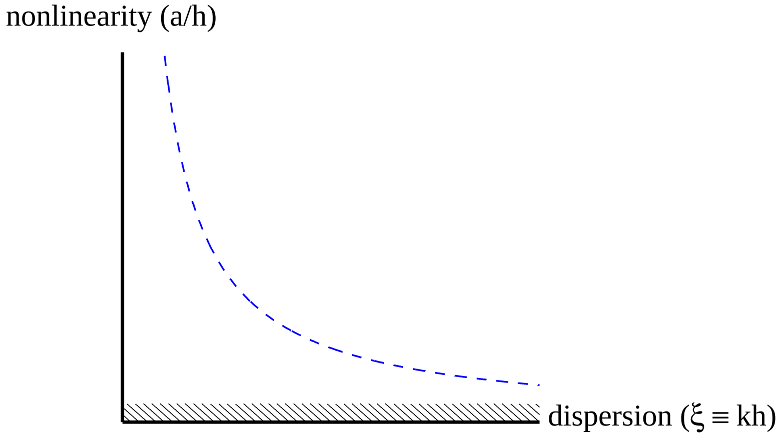
Thanks for your works!

Wave propagation models



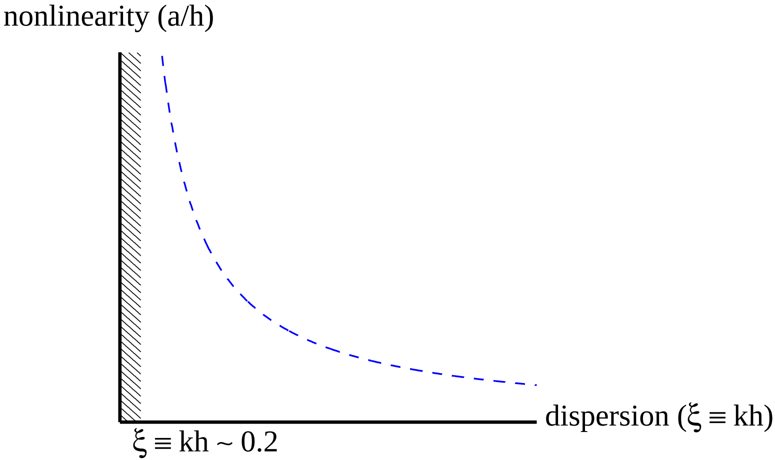
Wave propagation models

Linear theory: Airy, MSE, ...



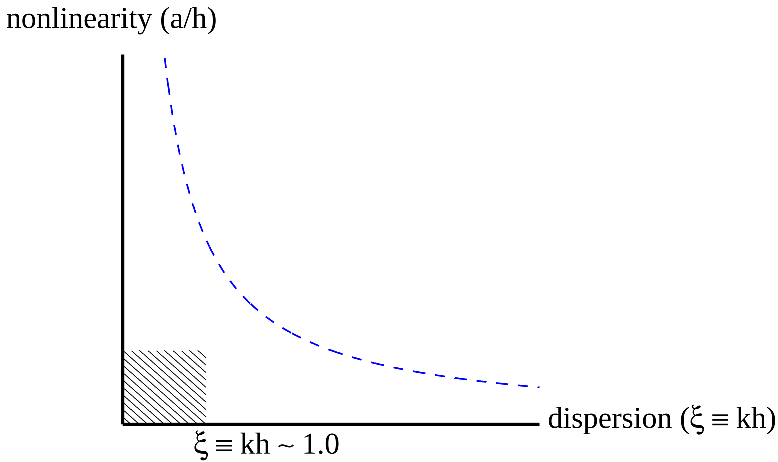
Wave propagation models

Nonlinear Shallow Water Equations



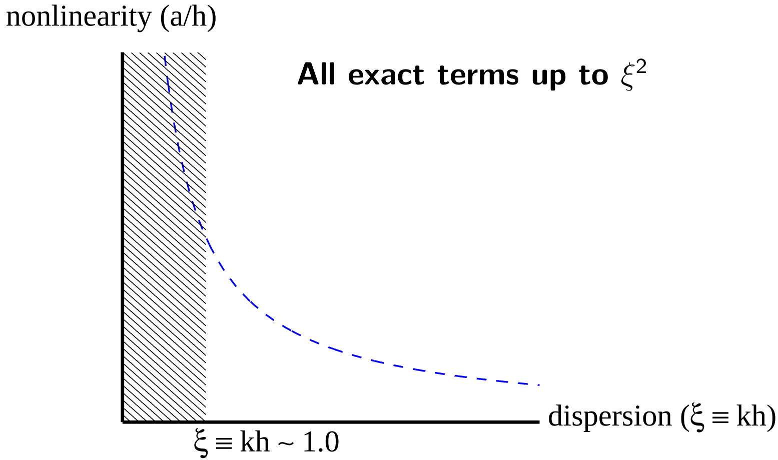
Wave propagation models

Boussinesq Equations (Peregrine '67)



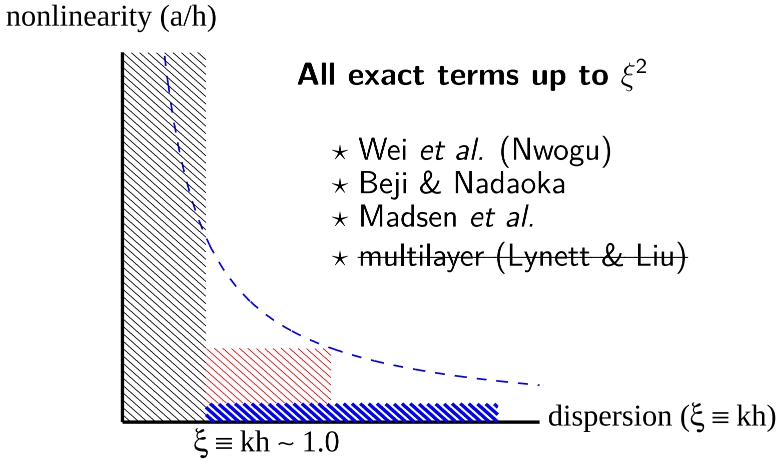
Wave propagation models

(fully nonlinear) Boussinesq-Type Equations (**BTE**)



Wave propagation models

(fully nonlinear) Boussinesq-Type Equations (**BTE**)



All exact terms up to ξ^2

- $\star$ Wei *et al.* (Nwogu)
- $\star$ Beji & Nadaoka
- $\star$ Madsen *et al.*
- $\star$ ~~multilayer (Lynett & Liu)~~

Low order BTE: exact up to $\mathcal{O}\left(\xi^2 \equiv (kh)^2\right)$

Low order BTE: exact up to $\mathcal{O}\left(\xi^2 \equiv (kh)^2\right)$

- introduce free coefficients (*e.g.*, $u_\alpha = u\left(z_\alpha h\right)$);

Low order BTE: exact up to $\mathcal{O}\left(\xi^2 \equiv (kh)^2\right)$

- introduce free coefficients (*e.g.*, $u_\alpha = u\left(z_\alpha h\right)$);
- all the models are exact up to $\mathcal{O}\left(\xi^2\right)$
  - Airy

$$\frac{\omega^2}{gk^2h} = \frac{\tanh \xi}{\xi} = 1 - \frac{\xi^2}{3} + \frac{2\xi^4}{15} + \dots$$

  - Wei *et al.* (based on Nwogu)

$$\frac{\omega^2}{gk^2h} = \frac{1 - (\alpha + 1/3)\,\xi^2}{1 - \alpha\xi^2} = 1 - \frac{\xi^2}{3} - \frac{\alpha\xi^4}{3} + \dots$$

the differences appear in higher order terms;

Low order BTE: exact up to $\mathcal{O}\left(\xi^2 \equiv (kh)^2\right)$

- introduce free coefficients (*e.g.*, $u_\alpha = u(z_\alpha h)$);
- all the models are exact up to $\mathcal{O}\left(\xi^2\right)$
  - Airy

$$\frac{\omega^2}{gk^2h} = \frac{\tanh\xi}{\xi} = 1 - \frac{\xi^2}{3} + \frac{2\xi^4}{15} + \dots$$

  - Wei *et al.* (based on Nwogu)

$$\frac{\omega^2}{gk^2h} = \frac{1 - (\alpha + 1/3)\xi^2}{1 - \alpha\xi^2} = 1 - \frac{\xi^2}{3} - \frac{\alpha\xi^4}{3} + \dots$$

  the differences appear in higher order terms;
- embedded order terms are important.

BTE: Wei *et al.* (Nwogu): linear analysis

BTE: Wei *et al.* (Nwogu): linear analysis

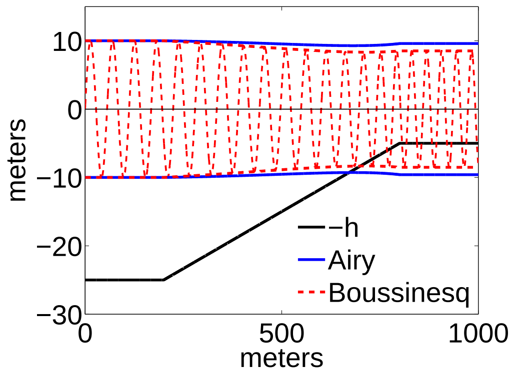
- wave celerity: for $\xi \leqslant 3$; error: 1%;

BTE: Wei *et al.* (Nwogu): linear analysis

- wave celerity: for $\xi \leqslant 3$; error: 1%;
- wave amplitude: for $\xi \leqslant 3$; error: 13% (?)

BTE: Wei *et al.* (Nwogu): linear analysis

- wave celerity: for $\xi \leqslant 3$; error: 1%;
- wave amplitude: for $\xi \leqslant 3$; error: 13% (?)



Why these errors in the amplitude?

Why these errors in the amplitude?

- BTEs introduce free coefficients chosen mainly to optimize celerity performance.

Why these errors in the amplitude?

- BTEs introduce free coefficients chosen mainly to optimize celerity performance.
- is wave amplitude less important than celerity?

Why these errors in the amplitude?

- BTEs introduce free coefficients chosen mainly to optimize celerity performance.
- is wave amplitude less important than celerity?
- is the problem evaluating the wave amplitude error?
  - for Boussinesq $c_g \equiv \partial\omega/\partial k$ **is not** so that
  $$A^2 c_g = ctt.$$
  if it was, it would be easy! ($A^2 c_g = ctt$ for Airy);
  - alternative analysis is “difficult”.

Why these errors in the amplitude?

- BTEs introduce free coefficients chosen mainly to optimize celerity performance.
- is wave amplitude less important than celerity?
- is the problem evaluating the wave amplitude error?
  - for Boussinesq $c_g \equiv \partial\omega/\partial k$ **is not** so that

$$A^2 c_g = ctt.$$

    if it was, it would be easy! ($A^2 c_g = ctt$ for Airy);
  - alternative analysis is "difficult".
- can we make it easier?.

Making it easier

Making it easier

- energy balance for Boussinesq yields

$$A^2 v_g = ctt,$$

where $v_g \neq \partial\omega/\partial k$ is a simple expression;

Making it easier

- energy balance for Boussinesq yields

$$A^2 v_g = ctt,$$

where $v_g \neq \partial\omega/\partial k$ is a simple expression;

- in fact $v_g = \partial\omega/\partial k$ to order $\mathcal{O}\left(\left(kh\right)^2\right)$, but

$$v_g = \partial\omega/\partial k + \mathcal{O}\left(\left(kh\right)^4, \ldots\right),$$

and we know that higher order terms matter!;

Making it easier

- energy balance for Boussinesq yields

$$A^2 v_g = ctt,$$

where $v_g \neq \partial\omega/\partial k$ is a simple expression;
- in fact $v_g = \partial\omega/\partial k$ to order $\mathcal{O}\left((kh)^2\right)$, but

$$v_g = \partial\omega/\partial k + \mathcal{O}\left((kh)^4, \ldots\right),$$

and we know that higher order terms matter!;
- evaluation of the amplitude error is easy;

Making it easier

- energy balance for Boussinesq yields

$$A^2 v_g = ctt,$$

where $v_g \neq \partial\omega/\partial k$ is a simple expression;

- in fact $v_g = \partial\omega/\partial k$ to order $\mathcal{O}\left((kh)^2\right)$, but

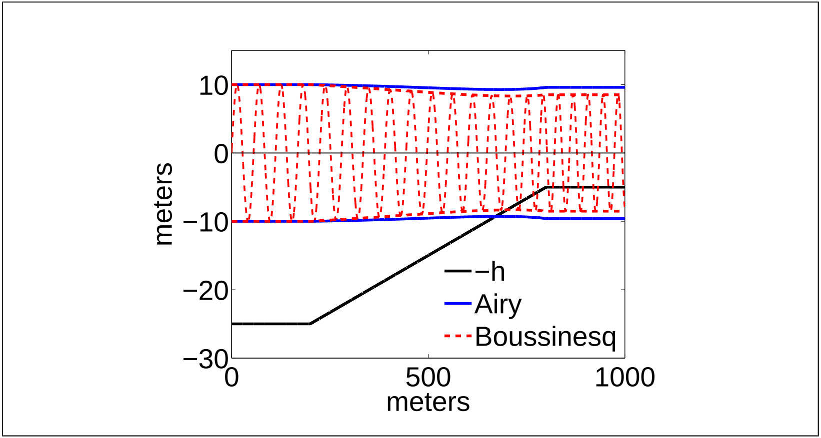
$$v_g = \partial\omega/\partial k + \mathcal{O}\left((kh)^4, \ldots\right),$$

and we know that higher order terms matter!;

- evaluation of the amplitude error is easy;
- let's us balance errors in celerity and amplitude.

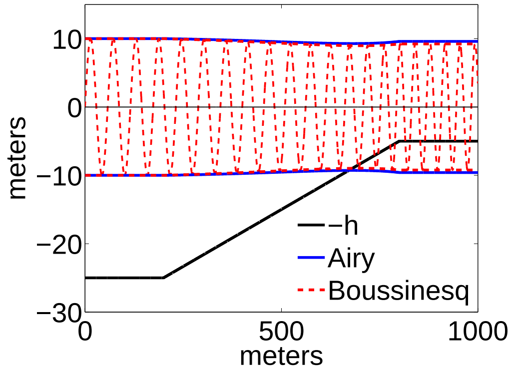
Wei *et al.* (Nwogu), original z_α

- celerity: for $kh \equiv \xi \leqslant 3$; error: 1.0%
- amplitude: for $kh \equiv \xi \leqslant 3$; error: 13%



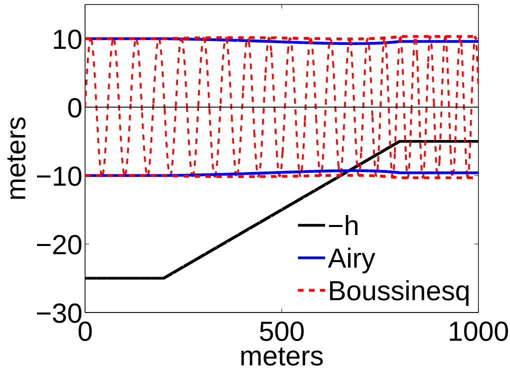
Wei *et al.* (Nwogu), new z_α

- celerity: for $kh \equiv \xi \leqslant 3$; error: ~~1.0%~~ $\rightarrow$ 4.5%
- amplitude: for $kh \equiv \xi \leqslant 3$; error: ~~13%~~ $\rightarrow$ 4.5%



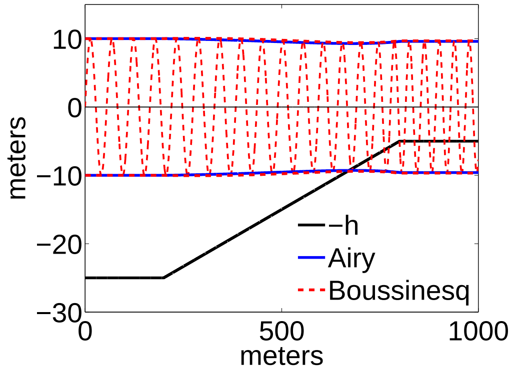
Beji & Nadaoka, original β

- celerity: for $\xi \leqslant 3$; error: 4.2%
- amplitude: for $\xi \leqslant 3$; error: 8.2%



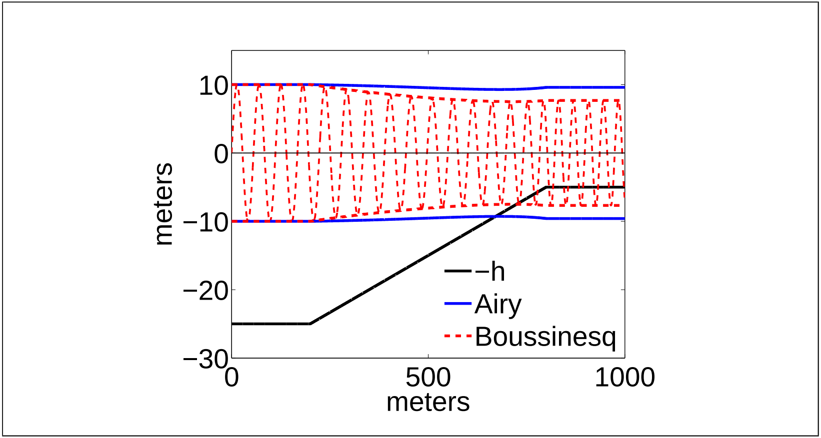
Beji & Nadaoka, new β

- celerity: for $\xi \leqslant 3$; error: ~~4.2%~~ $\rightarrow$ 1.8%
- amplitude: for $\xi \leqslant 3$; error: ~~8.2%~~ $\rightarrow$ 1.8%



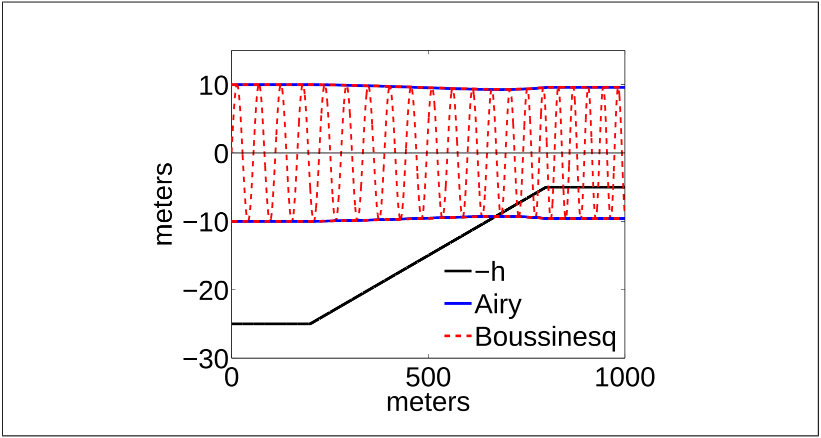
Madsen *et al.*, original parameters (5)

- celerity: for $\xi \leqslant 3$; error: 0.040%
- amplitude: for $\xi \leqslant 3$; error: 24%



Madsen *et al.*, new parameters (5)

- celerity: for $\xi \leqslant 5$; error: ~~0.040%~~ $\rightarrow$ 0.17%
- amplitude: for $\xi \leqslant 5$; error: ~~24%~~ $\rightarrow$ 0.17%



Final remarks

- a correct treatment of the wave amplitude error in Boussinesq-type Equations is compulsory;

Final remarks

- a correct treatment of the wave amplitude error in Boussinesq-type Equations is compulsory;
- the use of $\partial\omega/\partial k$ is an inadmissible shortcut, and the alternatives were *difficult*;

Final remarks

- a correct treatment of the wave amplitude error in Boussinesq-type Equations is compulsory;
- the use of $\partial\omega/\partial k$ is an inadmissible shortcut, and the alternatives were *difficult*;
- an energy balance analysis leads us to the correct group celerity;

Final remarks

- a correct treatment of the wave amplitude error in Boussinesq-type Equations is compulsory;
- the use of $\partial\omega/\partial k$ is an inadmissible shortcut, and the alternatives were *difficult*;
- an energy balance analysis leads us to the correct group celerity;
- wind waves with $T = 6\mathrm{s}$, allowing errors of 1%,

Wei *et al.* (Nwogu)	$h \sim 9\,m \rightarrow$	$h \sim 15\,m$
Beji	$h \sim 16\,m \rightarrow$	$h \sim 27\,m$
Madsen *et al.*	$h \sim 10\,m \rightarrow$	$h \sim 51\,m$

.

ELSEVIER

Wave Motion 33 (2001) 225–243

WAVE MOTION

www.elsevier.com/locate/wavemoti

Boussinesq-type equations with improved nonlinear performance

Andrew B. Kennedy*, James T. Kirby, Qin Chen, Robert A. Dalrymple

Center for Applied Coastal Research, University of Delaware, Newark, DE 19716, USA.

Received 19 November 1999; accepted 27 June 2000

Abstract

In this paper, we derive and test a set of extended Boussinesq equations with improved nonlinear performance. To do this, the concept of a reference elevation is further generalised to include a time-varying component that moves with the instantaneous free surface. It is found that, when compared to Stokes-type expansions of the second harmonic and fully nonlinear potential flow computations, both theoretical and practical nonlinear performance can be considerably improved. Finally, a special case of the extended equations is found to have properties which are invariant with respect to the still water datum. © 2001 Elsevier

Thanks for your works!